Self-collimation in liquid surface waves propagating over a bottom with periodically drilled holes

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Liquid surface waves propagating over a bottom with periodically drilled holes are studied experimentally and theoretically. Point source waves are generated in the center of the array of the periodically drilled holes in order to observe the evolvements of these waves. We successfully observe the self-collimation phenomenon in liquid surface waves. Band structures and constant-frequency surfaces are calculated, which can give a satisfactory interpretation of the experimental observations.

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Photonic crystals are artificial media with a spatially periodic variation in refractive index [1–3]. Multiple Bragg scatterings due to the introduced periodicity lead to the formation of complicated photonic band structures. Between photonic bands there may exist a photonic band gap within which electromagnetic waves cannot propagate. The existence of photonic band gaps can give rise to a variety of new physical phenomena, offering unprecedented opportunities to control and manipulate the propagation of electromagnetic waves [2,3]. On the other hand, the complicated photonic bands display strong dispersion and anisotropy, which are the origins of many interesting optical phenomena such as superprism [4], negative refraction [5], and self-collimation phenomenon [6–13]. Recently, the self-collimation phenomenon in photonic crystals has been the subject of intensive theoretical and experimental research [6–13] because of the resulting significant feature that a beam of electromagnetic waves can propagate almost without diffraction. Such an effect may offer another appealing approach to manipulate the propagation of electromagnetic waves, leading to many important applications such as the on-chip waveguiding, optical routing, bending, and splitting [6–13].

When propagating in periodic structures, liquid surface waves should be also modulated by multiple Bragg scatterings. The propagation of liquid surface waves is also characterized by band structures and there may exist a band gap [14–20], analogous to that in photonic crystals. Owing to the similar wave nature, many interesting phenomena found in photonic crystals can occur likewise for liquid surface waves propagating in periodic structures. For example, the present authors [21] demonstrated experimentally and theoretically the existence of a superlensing effect in liquid surface waves, which was originally proposed in photonic crystals [22]. Obviously, liquid surface waves have been manifest as excellent tools to demonstrate many interesting phenomena of classical waves. This is owing to the fact that not only the spatial distribution, but also the time-dependent evolution, of wave patterns can be directly observed. In fact, Bloch waves over a periodically drilled bottom, quasiperiodic Bloch-like waves over a quasiperiodically drilled bottom, and domain walls over a periodically drilled bottom with disorder, have been successfully observed in liquid surface wave experiments [23,24].

Liquid surface waves, e.g., water waves, are of great importance in both fundamental research and applications. The interesting phenomena found for electromagnetic waves in photonic crystals may likewise occur in liquid surface waves propagating in periodic structures. These unique phenomena in liquid surface waves may result in further physics and potential applications. As regards the self-collimation effect, it is, however, far from obvious whether this phenomenon would occur in liquid surface waves. In this paper, we present both the experimental observation and theoretical analysis of the self-collimation effect for liquid surface waves propagating over a bottom with periodically drilled holes. Point source waves are generated in the center of the array of the periodically drilled holes. Through the observation of the evolvements of the point source waves, we demonstrate the existence of the self-collimation effect in liquid surface waves propagating over the periodically drilled bottom.

The experimental setup is similar to that used in the observation of the superlensing effect in liquid surface waves [21]. Experiments are done in a vessel, whose bottom is a transparent glass. A flat methacrylate plate (24×24 cm^2) with periodically drilled holes is put on the bottom of the vessel. The thickness of the plate is 4 mm. The periodically drilled holes are arranged in a square lattice. The lattice constant is 18 mm and the radius of holes is 6.3 mm. The plate is then covered with a shallow liquid [25]. The depth of the liquid is about 0.85 mm above the plate. Thus, the drilled plate instead of the vessel bottom serves as the bottom for the liquid. A point source generator is placed in the center of the hole array of the plate in order to observe the self-collimation effect. The driven amplitude of the point source generator is kept rather small in order to avoid nonlinear effects. A halogen lamp is hung above the vessel (about 1 m). Projected images of liquid surface wave patterns can be visualized on the screen with the help of the mirror placed below the vessel. The detailed description of the experimental setup can be found elsewhere [21].

For an even bottom the point source generator will gen-

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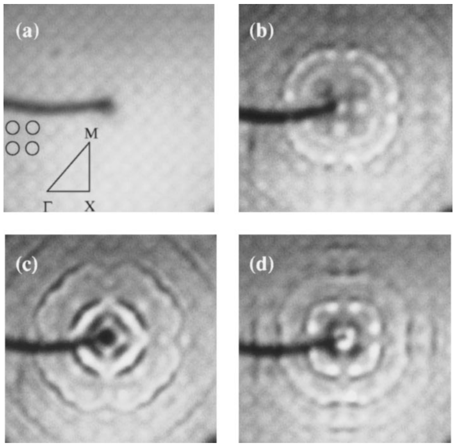

FIG. 1. Snapshots of the liquid surface wave patterns over a bottom with periodically drilled holes arranged in a square lattice for different driven frequencies. The point source generator is placed in the center of the hole array. (a) Calm liquid surface. No liquid surface waves are generated. The striplike dark image extending from the left to the center is the shadow of a tube of the point source generator. Brighter regions are the drilled holes (circles are guide for eyes). The irreducible Brillouin zone for the square lattice is also shown in the inset. (b), (c), and (d) show the wave patterns at different driven frequency $f=2.51$, 3.57, and 5.12 Hz, respectively.

erate a point source for small driven amplitude. However, the point source patterns should be modified if an uneven bottom is present. In the case of a bottom with periodically drilled holes the wave patterns are strongly modulated by the Bragg resonance [26]. Consequently, the waves should be a superposition of all possible Bloch waves with well-defined wave vectors. Figure 1 shows the snapshots of the liquid surface wave patterns. Liquid surface waves will spread out with time, forming ripples which can be clearly seen in the wave patterns. For low frequencies $f<3$ Hz, liquid surface waves can propagate along all directions and the ripples are continuous and circlelike [Fig. 1(b)]. As the frequency increases, the ripples might not be continuous and circular any more. Ripples can be broken in some directions, indicating that there is no wave propagation along these directions. This can be clearly seen from Fig. 1(c). The propagation of liquid surface waves is dominantly along the ΓM direction. Along the ΓX direction, however, there is no propagation of liquid surface waves. Figure 1(d) shows another case: the ripples are squarelike with rounded corners. The propagation of liquid surface waves is dominantly along the ΓX direction, while the propagation along the ΓM direction is rather small. It is interesting to note from Figs. 1(c) and 1(d) that the shape and length of the ripples along the dominant propagating directions are nearly kept invariable when spreading out, similar to the simulated results in photonic crystals [8]. This phenomenon is no other than the self-collimation effect. Thus, the experiments present the observation of the self-collimation effect in liquid surface waves.

To get some physical insight into the observed results, the band structures of liquid surface waves propagating over a bottom with a square array of periodically drilled holes are calculated. The computational methodology developed by our group was described in detail elsewhere [20], so only a brief description is given here. The propagation of liquid surface waves is dealt with a mild-slope equation [27,28], which can give a thorough description of the evolution of liquid surface waves over a bottom with varying topography. By introducing a new parameter $u\equiv cc_g/g$ (where g is the gravity's acceleration), the mild-slope equation can then be written in the form [20]

$$(\nabla\cdot u\nabla + k^2u)\varphi = 0, \qquad (1)$$

where $\nabla\equiv(\partial_x,\partial_y)$ is the horizontal gradient and φ is the complex horizontal variation of the velocity potential. The parameters $c=\omega/k$ and $c_g=d\omega/dk$ are the phase and group velocity, respectively. For inviscid liquids, the angular frequency ω and the local wave number k are related by the following dispersion relation [16]:

$$\omega^2 = gk\left(1+\frac{T}{\rho g}k^2\right)\tanh(kh), \qquad (2)$$

where h is the liquid depth, T is the liquid surface tension, and ρ is the liquid density. For the periodically drilled bottom, the liquid depth h is a spatially varied function. The introduced new quantity u can be viewed as the effective liquid depth, an important parameter to determine the existence of band gaps [20]. Within the framework of the plane-wave expansion method [20], the periodic functions u and k^2u are expanded by plane waves. By applying the Bloch theorem to the field φ, the problem to solve the mild-slope equation (1) casts into solving the following eigenvalue problem [20]:

$$\det|\mathbf{Q}(\mathbf{q},\omega)| = 0, \qquad (3)$$

where the elements of the matrix $\mathbf{Q}(\mathbf{q},\omega)$ are given by

$$Q_{\mathbf{G}',\mathbf{G}}(\mathbf{q},\omega) = [(\mathbf{G}+\mathbf{q})(\mathbf{G}'+\mathbf{q})]A_{\mathbf{G}'-\mathbf{G}} - B_{\mathbf{G}'-\mathbf{G}}. \qquad (4)$$

Here, $\mathbf{G}=(G_x,G_y)$ is the two-dimensional (2D) reciprocal lattice vectors, $\mathbf{q}$ is the wave vector in the first Brillouin zone, and $A_{\mathbf{G}}$ and $B_{\mathbf{G}}$ are the Fourier coefficients of the periodic functions u and k^2u, respectively. Band structures can then be obtained by solving the above equation. It should be noted that Eq. (3) is not a standard eigenvalue problem since the expansion coefficients $A_{\mathbf{G}}$ and $B_{\mathbf{G}}$ are dependent on ω.

When propagating over a bottom with periodically drilled holes, the dispersion of liquid surface waves is greatly modified as compared to that over an even bottom, characterized by complicated band structures as can be seen from Fig. 2. For the system considered, there exists no complete band gap. There are only some partial band gaps along certain directions. For example, along the ΓX direction, there is a partial band gap for frequency ranging from 2.97 to 3.95 Hz.

From the band structures many observed results shown in Fig. 1 can be accounted for. For low frequencies ($f<2.94$ Hz), there is no band gap for all directions. Therefore, liquid surface waves can propagate in any direction. For very low frequencies, the dispersion of liquid surface waves is nearly isotropic in all directions. The ripple patterns of liquid surface waves should be circlelike. The resultant wave patterns are similar to that shown in Fig. 1(b). As frequency

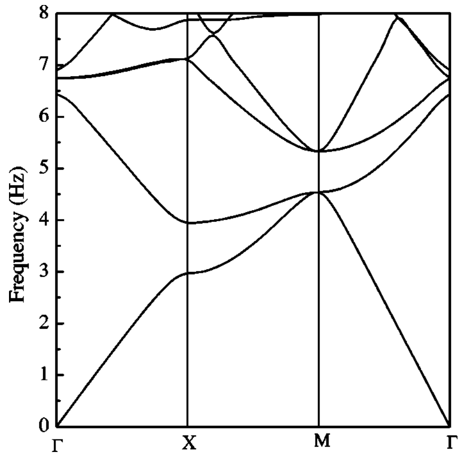

FIG. 2. Calculated band structure for liquid surface waves propagating over a bottom with an infinite square lattice array of periodically drilled holes. The parameters used in calculations are the same as in the experiment.

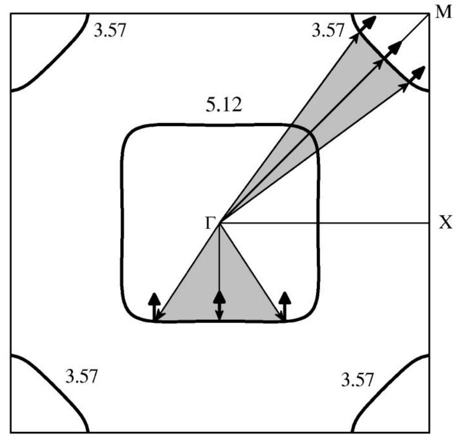

FIG. 3. Calculated CFSs (thick curves) in the first Brillouin zone for two frequencies $f=3.57$ and 5.12 Hz. The shaded regions show the wave vectors that make the group velocity vectors point to nearly the same direction, leading to the self-collimation effect. The thin and thick arrows denote the wave vector and the group velocity vector, respectively.

increases, anisotropic dispersion develops. The frequency range between 2.94 and 3.95 Hz is the partial band gap along the Γ*X* direction. As a result, propagation along this direction is not allowed. This can account for the fact that no ripples are observed along the Γ*X* direction at frequency $f=3.57$ Hz as shown in Fig. 1(c).

To better understand the observed self-collimation effect for the two frequencies $f=3.57$ and 5.12 Hz [Figs. 1(c) and 1(d), respectively], the constant-frequency surfaces (CFSs) for the two frequencies are also calculated, shown in Fig. 3. At the frequency $f=3.57$ Hz the CFS is centered at the *M* point, while that at $f=5.12$ Hz is centered at the Γ point. The CFS at $f=5.12$ Hz forms a square with rounded corners. At $f=3.57$ Hz it is also a square with rounded corners, but rotated by an angle of 45° with respect to the Brillouin zone. In both cases there exists some rather flat dispersion in CFSs. As we know, the direction of energy flow is determined by the energy velocity vector. It has been proved that the energy velocity vector is equal to the group velocity vector in photonic crystals [29]. It can be shown that this is also held for liquid surface waves propagating in periodic structures. The group velocity vector can be defined by $\mathbf{v}_g=\nabla_q\omega(\mathbf{q})$. From its definition, the group velocity vector is perpendicular to the CFS in the direction along which $\omega(\mathbf{q})$ is increasing [22]. For wave vectors that lie in these flat dispersion surfaces, waves should propagate nearly along the same direction, leading to the self-collimation effect [8]. As we can see from Fig. 3 the CFS at $f=3.57$ Hz is flat for some wave vectors in the vicinity toward the Γ*M* direction. Resultingly, wave propagation dominates along the Γ*M* direction and self-collimated waves can be observed along this direction. For frequency $f=5.12$ Hz the CFS is rather flat for wave vectors in the vicinity toward the Γ*X* direction. Similar self-collimation phenomenon can be obtained along the Γ*X* direction. Thus the CFSs can account for the observed self-collimated wave patterns in Figs. 1(c) and 1(d). It should be noted that self-collimation phenomena can occur only for very limited frequencies and directions, since the flat CFSs spreads over very limited wave vectors.

Interestingly, it can be seen from Fig. 3 that at $f=5.12$ Hz the direction of the wave vector is antiparallel to that of the corresponding group velocity vector. This phenomenon is referred to as the left-handed behavior, proposed by Veselago [30] in the late 1960s when discussing the properties of an imaginary material with a negative refractive index. The left-handed behavior has become a hot topic [31], since it gives rise to many unusual phenomena for electromagnetic waves, such as the reversal of Doppler effect, radiation pressure, and Cherenkov radiation. The left-handed behavior has been also found in photonic crystals [32]. These similar unusual phenomena may also occur in liquid surface waves when propagating in periodic structures, which are the subjects of future studies.

In conclusion, the propagation of liquid surface point waves over a bottom with periodically drilled holes has been studied experimentally and analyzed theoretically. Our work has presented both experimental and theoretical evidences of the existence of the self-collimation phenomena in liquid surface waves propagating in periodic structures.

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[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987); S. John, *ibid.* **58**, 2486 (1987).
[2] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, NJ, 1995).
[3] *Photonic Band Gap Materials*, edited by C. M. Soukoulis (Kluwer, Dordrecht, 1996).
[4] H. Kosaka, T. Kawashima, A. Tomita, M. Notomi, T. Tamamura, T. Sato, and S. Kawakami, Phys. Rev. B **58**, R10096 (1998).
[5] M. Notomi, Phys. Rev. B **62**, 10 696 (2000).
[6] H. Kosaka, T. Kawashima, A. Tomita, M. Notomi, T. Tamamura, T. Sato, and S. Kawakami, Appl. Phys. Lett. **74**, 1212 (1999).
[7] J. Witzens, M. Loncar, and A. Scherer, IEEE J. Sel. Top. Quantum Electron. **8**, 1246 (2002).
[8] D. N. Chigrin, S. Enoch, C. M. S. Torres, and G. Tayeb, Opt. Express **11**, 1203 (2003).
[9] L. Wu, M. Mazilu, and T. F. Krauss, J. Lightwave Technol. **21**, 561 (2003).
[10] C. H. Chen, A. Sharkawy, D. M. Pustai, S. Y. Shi, and D. W. Prather, Opt. Express **11**, 3153 (2003).
[11] X. Yu and S. Fan, Appl. Phys. Lett. **83**, 3251 (2003).
[12] D. W. Prather, S. Y. Shi, D. M. Pustai, C. H. Chen, S. Venkataraman, A. Sharkawy, G. J. Schneider, and J. Murakowski, Opt. Lett. **29**, 50 (2004).
[13] D. M. Pustai, S. Y. Shi, C. H. Chen, A. Sharkawy, and D. W. Prather, Opt. Express **12**, 1823 (2004).
[14] T. Chou, Phys. Rev. Lett. **79**, 4802 (1997).
[15] T. Chou, J. Fluid Mech. **369**, 333 (1998).
[16] M. Torres, J. P. Adrados, F. R. Montero de Espinosa, D. García-Pablos, and J. Fayos, Phys. Rev. E **63**, 011204 (2001).
[17] P. McIver, J. Fluid Mech. **424**, 101 (2000).
[18] Z. Ye, Phys. Rev. E **67**, 036623 (2003).
[19] X. Hu, Y. Shen, X. Liu, R. Fu, J. Zi, X. Jiang, and S. Feng, Phys. Rev. E **68**, 037301 (2003).
[20] X. Hu, Y. Shen, X. Liu, R. Fu, and J. Zi, Phys. Rev. E **68**, 066308 (2003).
[21] X. Hu, Y. Shen, X. Liu, R. Fu, and J. Zi, Phys. Rev. E **69**, 030201(R) (2004).
[22] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, Phys. Rev. B **65**, 201104(R) (2002).
[23] M. Torres, J. P. Adrados, and F. R. Montero de Espinosa, Nature (London) **398**, 114 (1999).
[24] M. Torres, J. P. Adrados, J. L. Aragón, P. Cobo, and S. Tehuacanero, Phys. Rev. Lett. **90**, 114501 (2003).
[25] This liquid is CFC-113 of the Dupont company, a popular solvent with a low viscosity $\upsilon = 0.70 \times 10^{-2}$ cm^2/s, very low surface tension $T = 17.3$ dyn/cm, and high density $\rho = 1.57$ g/cm^3. We use this liquid instead of water due to the fact that this liquid has a very low capillary length and small dissipation.
[26] C.-C. Mei, *The Applied Dynamics of Ocean Surface Waves* (World Scientific, Singapore, 1989).
[27] B. Li, Coastal Eng. **23**, 227 (1994).
[28] P. L.-F. Liu and I. J. Losada, J. Hydraul. Res. **40**, 229 (2002).
[29] K. Sakoda, in *Optical Properties of Photonic Crystals*, Springer Series in Optical Sciences Vol. 80 (Springer, New York, 2001).
[30] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[31] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000); R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[32] S. Foteinopoulou and C. M. Soukoulis, Phys. Rev. B **67**, 235107 (2003).